

**DIMENSION AND ANALYTICS IN SLOW GROUHY CRACK ON MATERIAL
VISCO-ELASTICITY**

PRO. HIMADRI VEKATESHWARA

(FACULTY IN MATELOGY SCIENCE DEPARTMENT IN UNIVERSITY URBANA CHAIMPAING, LLLINIOS)

ABSTRACT

Apparatus for studying slow crack growth during a polymer is described. A theoretical analysis is made public which relates the speed of growth to the stress intensity factor and material properties and demonstrates how the visco-elastic behavior of the polymer is chargeable for slow crack growth. The analysis predicts a intensity of stress intensity below which crack growth under static load mustn't occur. Experimental data on slow crack growth in two grades of PVC are obtained for comparison with theoretical predictions. Difficulties experienced with the gathering of stable growth data on these materials are associated with the occurrence of transient growth. The speculation is observed to explain slow growth in an exceedingly small range of stress intensity close to the edge only. At higher levels, experimental growth rates for both PVC grades are much under theoretical values and also the range of stress intensity giving stable growth is larger than that predicted. A modification to the theoretical analysis is taken into account which attempts to model more accurately the behavior of the plastic zone material, and preliminary calculations have indicated that this produces a much better description of experimental data.

INTRODUCTION

The observation of brittle failures in most engineering plastics after long times under load has stimulated research on the themes of crack initiation and growth in these materials. These subjects are difficult to review because the mechanisms involved are operative over long periods. Additionally, the specification of materials behavior is complicated by the very fact that crack initiation times and growth rates are influenced in complex fashions by a good range of factors. Polymer structure, temperature, the presence of alternating loads and particular chemical environments are some of the factors that are

observed to affect the character and kinetics of brittle fracture processes. Materials characterization thus involves an excellent deal of time consuming testing and a few considerations should run to the selection of test methods and analyses to make sure relevant and meaningful results.

The supply of procedures for interpreting data would reduce the number of testing needed to explain fracture behavior and enable the characteristics of a polymer to be presented in a form suitable for materials comparison or design. a considerable amount of labor has been reported within the literature on this subject, 1-3 but there's scope for an additional effort aimed toward handling the testing and analysis of a wider range of polymers under a greater type of loading situations. In this paper, a fracture mechanics analysis is made public for describing crack growth in visco-elastic material. It demonstrates that slow stable growth may be a consequence of your time dependence within the modulus of the material and relates crack growth rates to the applied stress intensity. The theory, therefore, appears attractive for the analysis and interpretation of slow crack growth experiments. It also relates the minimum or intensity level of stress intensity, below which crack growth shouldn't occur, to quantities that may be measured briefly or simple test procedures.

This threshold stress intensity factor would constitute a useful and relevant parameter for outlining the resistance of a polymer to slow crack growth. To assess the validity of this theory for describing slow crack growth during a polymer, theoretical predictions are compared with experimental crack growth data obtained on two grades of PVC. Certain difficulties can arise within the collection of stable growth data on these materials. Additionally, there are several features of the expansion behavior which aren't predicted by the idea. An interpretation of those features is presented, and, in light of this, improvements to the theoretical model are considered. In its present form, the proposed analysis is complicated but should be considered as a basis from which valid and more workable theories may be developed.

2. DERIVATION OF THE CRACK GROWTH LAW FOR A VISCOELASTIC MATERIAL

An outline of the steps resulting in a derivation of the expansion law is given during this section; the mathematical details of the analysis are presented elsewhere, 4s Figure 1 shows a crack of length $c(t)$ in an exceedingly polymer that's loaded within the direction x_2 normal to the plane of the crack. A line plastic zone sooner than the crack tip is depicted as one craze having a length $R(t) = a(t) - c(t)$ and is assumed nowadays to support a stress σ_p which is independent of your time and position within the zone. Under slow crack growth, the crack advances through the zone and new plastically deformed material are incorporated simultaneously into the zone at its tip. the speed at which work is completed on the plastic zone by the strain field within the remainder of the polymer is then balanced by the speed at which energy is dissipated within the zone through plastic deformation and crack growth. Equating these quantities yields the subsequent local energy balance equation.

$$\int_{c(t)}^{a(t)} \sigma_p \frac{\partial \Delta u_2(x_1, t)}{\partial t} dx_1 = 2\Gamma \dot{c}(t) \quad (1)$$

where $\dot{c}(t)$ is the speed of growth, $\Delta u_2(x_1, t)$ is the discontinuity of the component u_2 representing the displacement of the opposite surfaces of the zone, and Γ is the fracture energy per unit area. The quantity Δu_2 may be expressed in terms of x_1, t, R and the tensile creep function $J(t)$ of the polymer. The magnitude of the stress field around

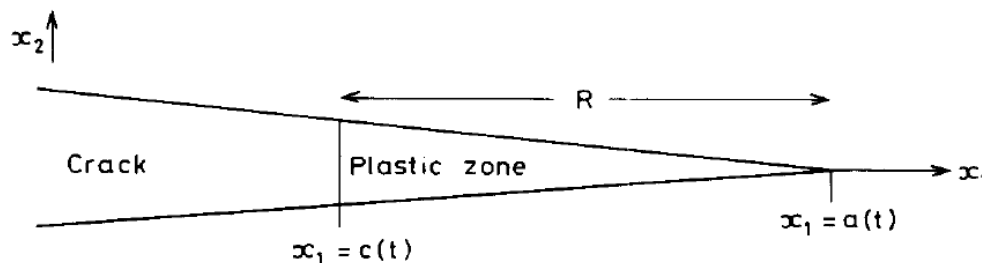


Fig. 1. Schematic diagram of the region around the tip of a crack in a material.

the crack increases as the plastic zone is approached and it must be bounded at the point $x_1 = a(t)$, $x_2 = 0$. This requirement leads to the following relationship for the zone length R (valid only for the line plastic zone model under discussion)

$$R(t) = \frac{\pi K(t)^2}{8 \sigma_p^2} \quad (2)$$

where $K(t)$ is the stress intensity factor and defines the magnitude of the stress field around the crack tip. $K(t)$ can be determined from a knowledge of the applied load P and the crack length c through relationships of the form

$$K(t) = P c(t)^{1/2} Y \quad (3)$$

The parameter Y is governed by the size and geometry of the loaded material and the crack length. Expressions exist⁶ enabling Y to be calculated for a wide range of common geometries. If the relationships for Δu_2 and R are substituted into eqn. (1), an expression for the crack speed \dot{c} may be derived in terms of K and the material properties $J(t)$, Γ and σ_p . This is the crack growth law for the polymer.

The theory demonstrates that slow crack growth is a direct consequence of the viscoelastic behaviour of a material and stipulates that growth will only take place if the stress intensity lies between a minimum value K_{TH} and a maximum value K_c . If the compliance function $J(t)$ approaches a limiting value $J(\infty)$ at long times, then the threshold stress intensity is given by

$$K_{TH}^2 = \frac{2\Gamma}{J(\infty)} \quad (4)$$

The upper bound K_c , for which $\dot{c} \rightarrow \infty$ as $K \rightarrow K_c$, is governed by the zero-time or elastic limit $J(0)$ to $J(t)$ and is given by

$$K_c^2 = \frac{2\Gamma}{J(0)} \quad (5)$$

Equations (4) and (5) imply that the value of K_{TH} may be simply derived from a short-term fracture experiment to determine K_c and a knowledge of the creep function for the polymer. In materials which do not exhibit a long-term limit to $J(t)$, no threshold to crack growth is predicted. The implication then is that, if a long-time value for $J(t)$

$$K = \frac{PY\sqrt{c}}{BW} \quad (6)$$

where Y is a function of c/W . For $0.3 < (c/W) < 0.7$, it is stated that

Y is given to an accuracy of 1% by

$$Y = 29.6 - 186\left(\frac{c}{W}\right) + 656\left(\frac{c}{W}\right)^2 - 1017\left(\frac{c}{W}\right)^3 + 639\left(\frac{c}{W}\right)^4 \quad (7)$$

the crack increases as the plastic zone are approached and it must be bounded at the point $X_x = a(t)$, $X_e = 0$. This requirement leads to the following relationship for the zone length R (valid only for the line plastic zone model under discussion)

where $K(t)$ is the stress intensity factor and defines the magnitude of the stress field around the crack tip. $K(t)$ can be determined from a knowledge of the applied load P and the crack length c through relationships of the form

The parameter Y is governed by the size and geometry of the loaded material and the crack length. Expressions exist 6 enabling Y to be calculated for a wide range of common geometries. If the relationships for Au^2 and R are substituted into Eqn. (1), an expression for the crack speed d may be derived in terms of K and the material properties $J(t)$, F and σ_p . This is the crack growth law for the polymer.

The theory demonstrates that slow crack growth is a direct consequence of the viscoelastic behavior of a material and stipulates that growth will only take place if the stress intensity lies between a minimum value K_{a-i} and a maximum value K_c . If the compliance function $J(t)$ approaches a limiting value $j(\infty)$ at long times, then the threshold stress intensity is given by

Equations (4) and (5) imply that the value of K_{TH} may be simply derived from a short-term fracture experiment to determine K_{ϕ} and knowledge of the creep function for the

polymer. In materials that do not exhibit a long-term limit to $J(t)$, no threshold to crack growth is predicted. The implication then is that, if a long-time value for $J(t)$

were substituted into Eqn. (4), the calculated value of KT_n would correspond to crack growth at a very slow speed which, for most practical purposes constitutes a threshold level. To assess the validity of this theory for describing slow crack growth in a polymer, predicted growth laws have been compared with experimental data obtained for two grades of PVC.

3. MATERIALS

Two transparent grades of PVC were obtained from ICI PLC. One of these was extruded sheet of about 6-4 mm thickness manufactured under the trade name Darvic®. The other was compression molded material, also about 6.4mm thick, having a chemical specification closer to that used in engineering grades of PVC with the change of ingredient necessary to render the material transparent. These materials are designated PVC-D and PVC-H, respectively. Material H showed no birefringence. In the sheet of D material, the in-plane birefringence varied with position in the sheet from 0 to about 2×10^{-4} . This birefringence could only be removed by heating above 180°C. It was concluded, therefore, that its origin was due to molecular orientation maintained at temperatures above T_g by microcrystalline regions acting as cross-links in the molecular chain network. No thermal treatments were applied to the samples used for the crack growth studies. All materials are therefore as-received, and the majority of data has been obtained at least 2 years after procurement.

4. EXPERIMENTAL

In this section, apparatus is described for obtaining data on slow crack growth. The theoretical growth rate predictions require information on the creep compliance function $J(t)$ for the polymer, the fracture energy F , and the plastic zone stress σ_p . It is proposed to determine F from the measurement of K_{Ic} , using Eqn. (5), and σ_p from measurement of R

using eqn. (2). Test procedures are also described for evaluating these quantities experimentally.

4.1. Crack growth studies

The objective of the crack growth studies is to determine how the velocity of slow stable crack growth is related to the stress intensity. Experiments are carried out on specimens in which a crack has been artificially introduced and for which the stress intensity at the tip of the crack arising from an applied load can be calculated.

4.1.1. Specimen geometry

All measurements of crack growth were made on compact tension specimens having the geometry depicted in Fig. 2. For a specimen of width W and thickness B having a crack of length c and under a load P , the stress intensity K is calculated thus 6 .The notch is introduced by fly-cutting both surfaces so that the cuts meet in the centre of the specimen. The preparation of a sharp crack may be readily achieved in PVC by fatigue loading. The crack is initiated by inserting a small razor cut at the tip of the notch and fatigue loading until the crack has growth out of the chevron produced by the fly-cutting.

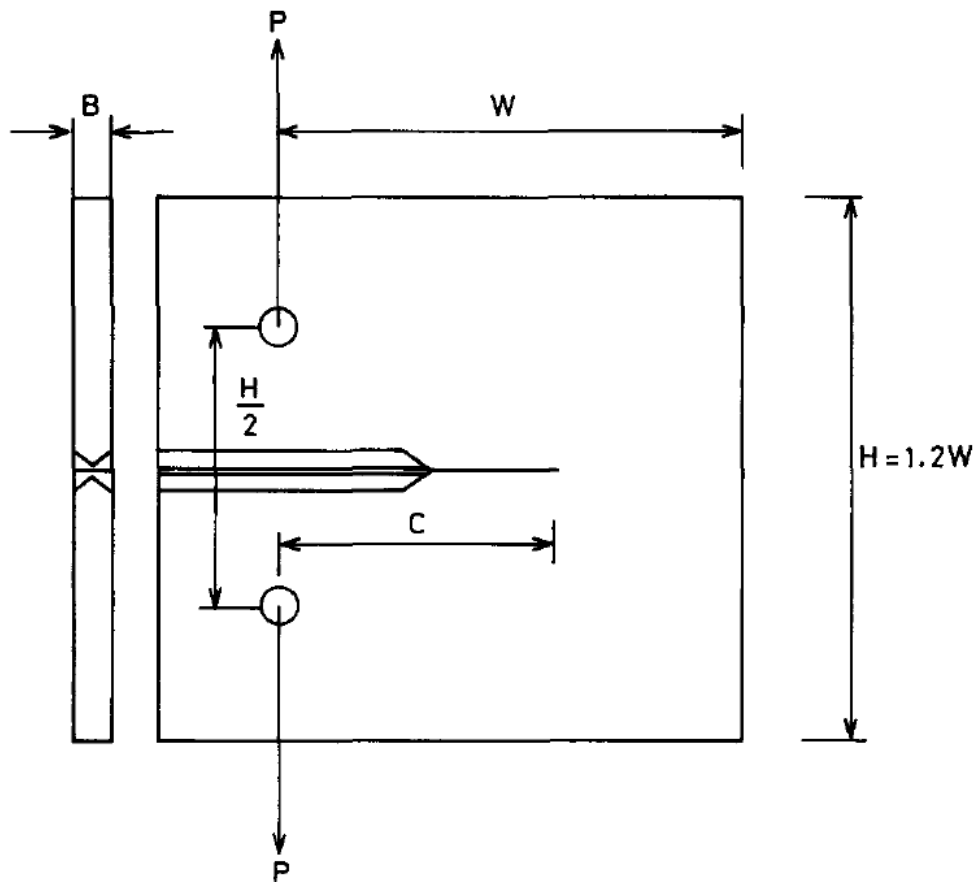


Fig. 2. Compact tension specimen geometry for crack growth studies.

4.1.2. Apparatus Crack growth investigations

under constant load have been carried out on simple apparatus consisting of a cross-head from which the compact tension sample is suspended via one of the pin holes in the sample. The load P is generated by the application of weights to a pan hanging from the other hole. The crack length is determined using a low-power travelling microscope viewing the crack tip at an angle of about 45° to the plane of the crack. Changes in crack tip position with time can usually be recorded to ± 0.01 mm using a micrometer eyepiece or the vernier scale on the microscope.

To eliminate random movements of the test piece with respect to the foundation of the microscope, it is necessary to record the position of

a reference mark on the specimen (usually a fine scratch) alongside each crack tip reading. Since the load remains constant, the stress intensity will increase as the crack advances. Reasonable accuracy in crack speed measurements can, however, usually be achieved for small increases of crack length (~ 0.5 mm). These will give rise to only small changes in K , so K is essentially constant during the crack speed determination. The load, and hence the stress intensity, is then raised incrementally in order to obtain further crack speed data over the range of K responsible for stable crack growth. One disadvantage of constant load testing arises when there is a possibility that the crack growth rate might increase substantially as this could lead to specimen failure before measurements of crack position can be made and the load reduced. This situation arises with the testing of materials exhibiting transient growth as discussed in Section 5. It is then more suitable to test under constant applied displacement so that as the crack advances the stress intensity decreases. For this purpose a simple loading assembly has been constructed whereby the specimen is connected in series with a force transducer to a threaded shaft which enables a static displacement to be applied. The other end of the sample can be connected to a rigid foundation.

A simple fatigue facility becomes available if this foundation is replaced by a source of dynamic displacement achieved, for example, by means of an eccentric shaft driven by an electric motor. The apparatus then enables sharp cracks to be prepared by dynamic loading. The amount by which the stress intensity falls with crack growth in a constant displacement test can be reduced by inserting an elastic (metal) ring in series with the specimen. As the compliance of the specimen increases due to crack advance, the displacement across the specimen will increase by an amount dependent upon the magnitude of the compliance of the ring in comparison with that of the specimen. It is possible to select a compliance for the ring such that the stress intensity is held almost constant for a substantial increase in crack length.

4.2. Measurement of plastic zone length

The crack surface appears very smooth in a specimen prepared by low-amplitude fatigue cycling ($K_{max} < 0.2 \text{ MN m}^{-3/2}$). If a load is then applied giving rise to a stress intensity above the fatigue maximum but below that which causes transient crack growth, then the size of the plastic zone resulting from this load may be measured since it appears rougher. The zone length R is observed to grow with time under load. Data on the increase of zone length with time for both grades of PVC are recorded in Figs 3 and 4 at different values of the stress intensity. Prior to each set of measurements, the sample was fatigued to prepare a new crack tip having a relatively small plastic zone again.

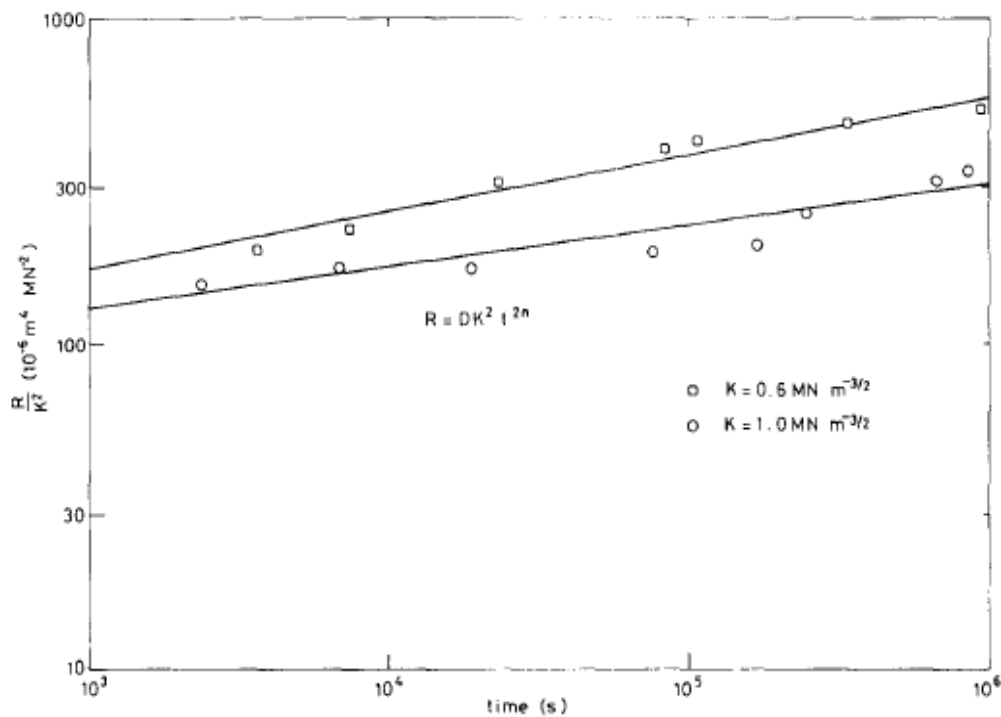


Fig. 3. Variation of plastic zone length R with time t under load plotted on logarithmic axes of R/K^2 against t . Darvic material.

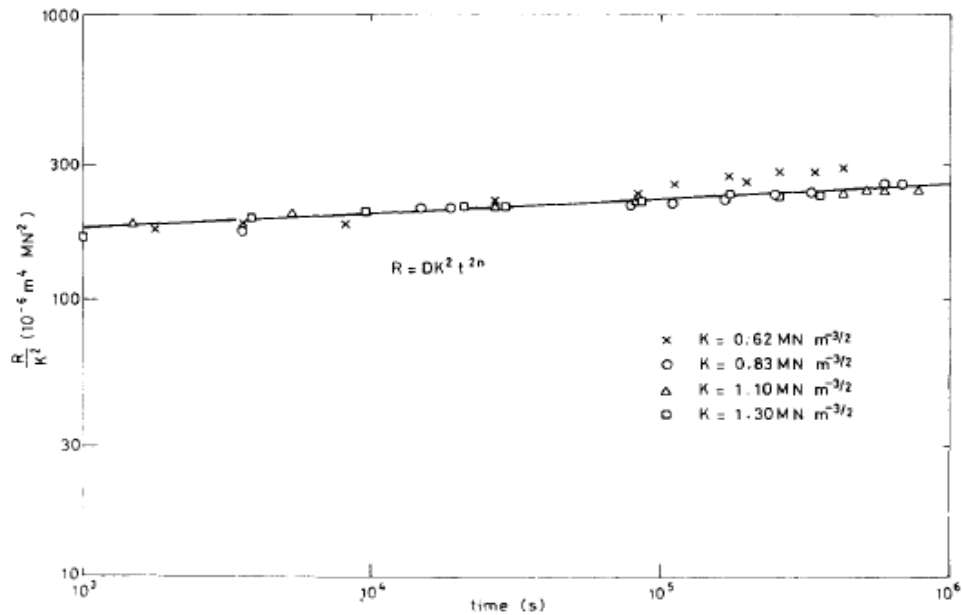


Fig. 4. Variation of plastic zone length R with time t under load plotted on logarithmic axes of R/K^2 against t . PVC material H.

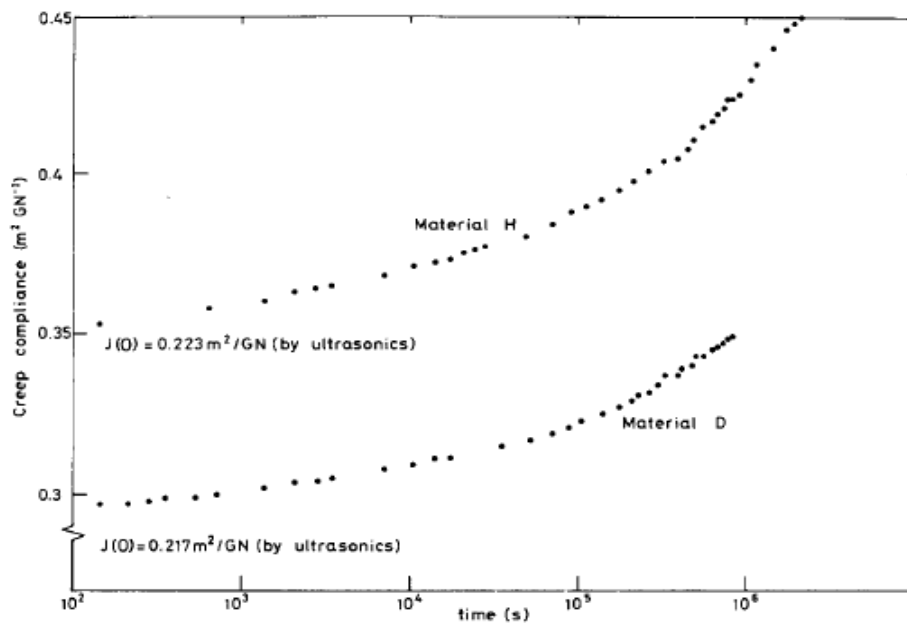


Fig. 5. Creep compliance against log (time) for both PVC materials.

creep compliance $J(t)$ was deduced from the equation

$$J(t) = \frac{\varepsilon(t)A}{P} \quad (8)$$

4.3. Determination of creep compliance

Creep studies were carried out on rectangular strips of uniform cross-sectional area A held under constant tensile load P . The longitudinal strain $e(t)$ was measured with time under load,

5. CRACK GROWTH OBSERVATIONS AND RESULTS

Crack growth in both PVC materials could be achieved over a wide range of applied stress intensity. This range is indicated in Fig. 6 along with the loading conditions necessary to induce growth at a particular K level. Under cyclic loading at a frequency of about 1 Hz, slow stable growth can be achieved at dynamic K amplitudes of around $0.25 \text{ MN m}^{-3/2}$. This loading schedule was used to generate a sharp crack in a specimen prior to static load experiments. If a stress intensity in the region of $1 \text{ MN m}^{-3/2}$ was applied fairly rapidly (within 10 s) to a sample in which a fatigue crack had been induced, crack advance in both PVC grades would usually follow an incubation period (about 1 h).

This growth was a transient phenomenon and, as long as the applied K was not too high, the growth rate would decrease to zero. Transient growth could be substantially reduced if the stress intensity level was reached by loading in small increments with around 24 h separating each load application. Under these conditions, comparable K values could be achieved with only marginal transient crack growth taking place and only then in the central region of the sample. Consequently, the crack tip profile would become increasingly curved after each load application. If this incremental loading procedure was continued, slow stable crack growth could be induced in material H at stress intensities of about $1.3 \text{ MN m}^{-3/2}$ and above.

In the Darvic material, slow stable crack advance was unusual even at K values approaching $2 \text{ MN m}^{-3/2}$. In this material, the plastic zone, which at these K levels appeared to consist of a bunch of crazes, was apparently larger, both in length and thickness, at the surfaces of a sample than near the centre.

RESULT

These data were obtained on four different samples, three possessing grooves and one un grooved. Each set of data exhibits a sharp onset of crack growth. With three of the samples, this is followed by a range of stress intensity for which the growth rate was approximately constant after which the speed increased rapidly again before failure of the sample. In the other sample, this period of sustained slow growth was absent. Data at growth rates higher than those recorded here ($>10^{-5}$ mm s⁻¹) could not be obtained. It is believed that once these speeds were reached the crack would accelerate even under constant or slowly reducing K and sample failure would soon result. The apparent scatter in data arises since the K range for stable growth is slightly different for each sample. The dotted line in demonstrates the trend in behavior and refers to one of the samples. No attempt has been made to allow for the presence of the grooves in the calculation of K values. Data showing the variation of crack speed with stress intensity for material H. These values were obtained on two samples represented by different symbols. Data at the lower values of K were obtained on one of the samples by progressively decreasing the stress intensity until the crack stopped growing. The scatter in velocity is real and reflects a variation that could be obtained daily. The data generally represent the average value for several day's of growth.

7. DISCUSSION

It is apparent from a comparison of the theoretical crack growth predictions with experimental data that, at best, the theory is only able to describe crack growth near the threshold. At higher values of K , the crack speed rises less sharply than predicted leading to slow growth over a wider range of K than implied by means. (4) and (5). The reasons behind this poor agreement could stem from inadequacies in the model when applied to PVC or limitations on the validity of the experimental data presented here. Some of the possible causes are considered later in this section but an attempt is made first to give an interpretation of crack growth behavior in the materials studied.

7.1. An interpretation of the salient features

It is clear that the resistance to crack growth of the grades of PVC studied increases with time under load and, in comparison with the theory, with increasing load. This behavior would appear to be associated with the development of the structure of the crazed the material in the zone ahead of the crack. This material is known to be highly non-linear and viscoelastic, 7 and the fact that structural changes that take place over a time scale of days is demonstrated by the results of measurements on plastic zone length. The larger size of the zone at the sample surfaces, coupled with the reluctance sometimes for the crack to advance at the surface, also bears evidence to the belief that structural changes in the zone can lead to a situation in the sample which is more resistant to crack advance.

The properties of the zone could contribute directly to the enhanced toughness or indirectly through a change in the stress field immediately ahead of the crack. The onset of transient growth, whereby the crack advances when the load is applied relatively rapidly, would take place before the zone had reached an equilibrium structure. If the crack tip speed under transient growth were such that the age of the zone ($-R/f$) was greater than the time required for the zone to develop a more resistant structure, then the crack would decelerate. The converse the situation, in which a weak zone structure is established or maintained under high crack speeds, could go some way towards explaining the unstable growth observed in the Darvic material at the higher levels of K in slow growth experiments and constant deformation rate tests.

7.2. Stable growth and the theoretical model

The theoretical growth laws are appropriate only for a crack having a single craze at the crack tip as depicted schematically in Fig. 1. Observation suggests that, whilst a single craze may exist at small values of the stress intensity ($K < 0.3 \text{ MN m}^{-3/2}$), multiple crazing is observed at the higher values used in these experiments. The relationship used for the displacement discontinuity Au_2 is substituted into the energy balance eqn. (1) will be different in this multiple crazing situation. Further improvements to the model should also be made regarding its ability to describe plastic deformation ahead of the

crack. It is known that craze material exhibits the behavior of a non-linear, viscoelastic solid implying that the quantity σ_p in Eqn. (1) should be replaced by a constitutive relationship demonstrating the dependence of σ_p upon deformation history. Although the experiment has indicated that \dot{c}_{rp} can be described by Eqn. (10), this was shown in Figs 3 and 4 to be true for a limited range of K well below those levels for which experimental and theoretical crack growth data depart. The time dependence in plastic zone behaviour may also lead to a relaxation of the stress field around the crack tip (blunting?) so that its magnitude local to the tip may no longer be characterized by the calculated stress intensity value given by Eqn. (6). A modification to the model has been considered, and preliminary calculations have shown that, by a suitable choice of a constitutive law for the craze zone, a crack growth curve resembling that obtained experimentally for material H can be predicted. The larger gradient at lower stress intensities is predicted including a threshold value for K . If the effective fracture energy is calculated using this model, it is found to be related to the magnitude of K and the time. The model is therefore consistent with the interpretation of material behavior.

REFERENCES

1. Williams, J. G. (1978). *Advances in Polymer Science*, **27**, 67–120.
2. Andrews, E. H. (Ed.) (1979). *Developments in Polymer Fracture—1*, London, Applied Science Publishers Ltd.
3. Bucknall, C. B. (1977). *Toughened Plastics*, London, Applied Science Publishers Ltd.
4. McCartney, L. N. (1979). *Int. J. Fracture*, **15**, 31–40.
5. Dean, G. D., McCartney, L. N., Cooper, P. M. and Golding, S. L. (1982). National Physical Laboratory Internal Report No. DMA (A) 57, Teddington.
6. Rooke, D. P. and Cartwright, D. J. (1976). *Compendium of Stress Intensity Factors*, London, HMSO.
7. Kambour, R. P. (1973). *J. Poly. Sci.*, **7**, 1–154.
8. Fernando, P. L. and Williams, J. G. (1980). *Poly. Engng. Sci.*, **20**, 215–20.
9. Pitman, G. L. and Ward, I. M. (1979). *Polymer*, **20**, 895–902.